Color Matching – Subjective vs Scientific

INTRODUCTION

When a building is damaged, and repairs are necessary; the damaged materials will either be restored or replaced. The restored or replaced materials should match the existing materials for an aesthetically pleasing structure. If the building is modern, the goal will be to bring the building back to a pre-loss condition so that people viewing the structure should not be aware that there had been a damaging event.

In general, original materials should be used if available. If original materials are available and acceptable by code, the solution is as simple as removing and replacing the damaged materials.

If the building is historical or if the building is outdated, the original materials may not be available. Original material may not be suitable for replacement because building techniques have changed as technology and codes have changed.

When original materials are not an option, damaged materials should be reasonably replaced with like-kind and like-quality. "Reasonably" is addressed at the end of this paper. Like-kind and like-quality are tangible.

Like-kind means the material looks and responds the same way as the original material. The replacement material should have the same dimensions, color, pattern, and response. There are numerous issues that can get in the way of finding like-kind materials; too many to list in this document.

This document addresses one aspect of selecting a replacement material – COLOR MATCHING. Color is a property of an object. Specifically, color is the sensation on the eye as the object reflects light.

Example 1: Color Matching Methods

Problem: A building with gray siding had a fire. A small portion of the siding was damaged. The original siding was no longer available. A proposed replacement product was presented. The question to be answered is, "Are the following 2 colors considered a close enough color match?"

Method 1: Human Observation - This method is simply a visual evaluation. The colors are placed side by side and a person visually compares the two colors. Based on the visual examination, the observer(s) would make a determination.

For the above colors, the correct conclusion should be, "No, the colors are not a match, the color on the right is a lighter gray."

Method 2: Color Matching Analysis – This method is a repeatable, scientific approach of color matching which includes measuring the colors with a spectrometer to obtain the numerical representations of color (L*, a*, b*). The computed differences in the colors are represented as ΔL, Δa, and Δb along with an overall ΔE to compare the two colors. The following values were calculated for the two above colors:

$L_1^* = 50.24$ $L_2^* = 62.41$

$a_1^* = -0.96$ $a_2^* = -0.39$

$b_1^* = 0.28$ $b_2^* = 1.05$

$\Delta E = 11$ which is > 2.0; therefore, the colors are mathematically dissimilar.

This paper introduces the science behind color matching and presents the calculations needed for this analysis.

Method 1 – Human Observation

This method is highly dependent on human perception. It is often uncontrolled. In certain situations, there would be nothing wrong with a simple visual determination. Visual human observations is normally only acceptable when one of the two is true:

1. The difference between two colors may be so far apart that it is clear to all involved that the colors do not match each other.
2. The difference between the two colors are so similar that no distinction can be made.

In these situations, there is no reason for further evaluation. The problem comes when there are opposing answers to the matching question and money is involved.

There are numerous uncontrolled variables in visual observations. The following is an incomplete list of some of the limitations of solely relying on human observation:

- Often, the two colors are not available for side by side viewing. In many cases, materials cannot be placed next to each other until it is too late and the materials have been installed.
- There can be a lack of control on the lighting. Colors are dependent on the source of light. Midday sunlight is different from morning or evening light. Outside sunlight is different from indoor artificial light. Objects reflect different color tones when it is cloudy or sunny.
- Colors can change with a change in the viewing angle.
- A person's background has an effect on viewing colors. An interior designer will respond differently to differences in color than a disinterested casual observer.
- The human eye has limitations from person to person. It has been shown that the age and mood of a person can affect color perception.

Method 2 – The Scientific Approach

Colors can be measured with a simple inexpensive spectrometer. Ancient artists and the paint industry have long been mixing and matching colors before the science was fully developed. Today, there are simple handheld devices that can very accurately identify color and light.

Every color can be represented by a set of numbers within a specified color space. Whether two colors match or not can be reduced to purely a scientific evaluation rather than a subjective human observation. The straightforward process to follow:

1. Obtain the number representation of each color.
2. Convert the color numbers to a specific color space such as the CIE Lab color space.
3. Calculate the mathematical difference between the two colors (ΔE).
4. Compare the difference with an acceptable standard (such as $\Delta E < 2.0$). This becomes the pass/fail criteria.

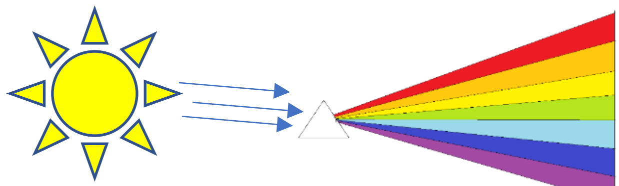
The remaining portion of this paper is dedicated to explaining the science behind this approach of color matching. But first, we need to understand the science of light to understand color.

Electromagnetic Spectrum & Color

The visible color spectrum is a small portion of the electromagnetic spectrum. The full spectrum includes:

Gamma Rays	X-rays	Ultraviolet Light
Infrared Light	Microwaves	Radio Wave

When invisible light passes through a prism, it is separated according to wavelength. The colors of the spectrum can be seen on the back side of rain drops.



Visible light has wavelengths from 360nm to 780nm.

Luminance

Luminance is a noun. It is the intensity of light emitted from a source. This can be measured.

Illuminance

Illuminance is the measure of the intensity of light on an area.

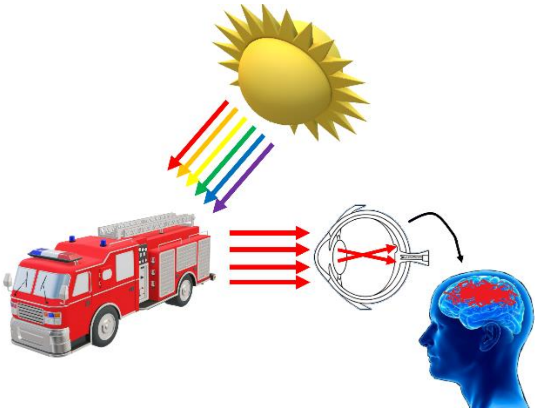
Reflection & Absorption

When light is pointed onto an object, certain wavelengths will be absorbed while other wavelengths will be reflected. This is a characteristic of the object. The reflected wavelengths combine to give a perception of color.

Color needs four things to be realized:

1. A light source
2. An object
3. A sensor (the eye)
4. A computing device (the brain)

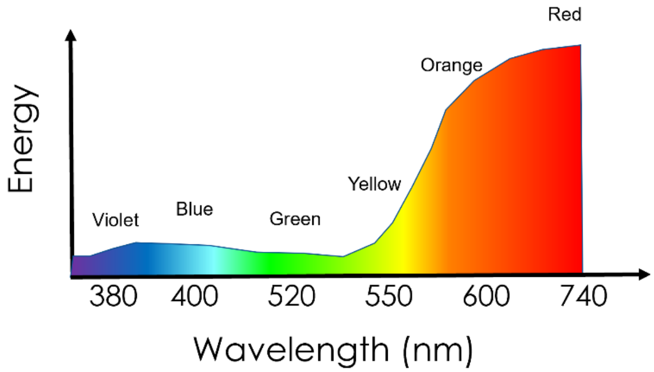
The image below shows the process of human perception of color. The red firetruck absorbs lower frequencies and reflects the higher frequencies of electromagnetic waves.



The eyeball senses the higher red frequencies and sends the information to the brain. The brain computes all the colors and makes a three dimensional image of the entire observed space. The brain takes the images and sends the information at lightning speed to the other areas of the brain to process emotions, reactions, memories, and other responses.

Instrumentation

Scientist have developed instruments to replace the eye and the brain. Color sensing instruments measure the reflected light in terms of wavelengths and convert the values to color terminology. Spectrometers are able to identify the wavelengths percentages of reflected light. Below is a representation of the wavelength distribution reading of the reflected light from a red surface.



As can be seen from the graph, the red block may have the appearance of red, but it has other wave lengths as well. In the real world, the reflective light of an object is never just one wavelength. It is a mixture of other wavelengths.

Human Perception – Trichromatic Vision

Color is a property of an object; however, the perception of color involves three things:

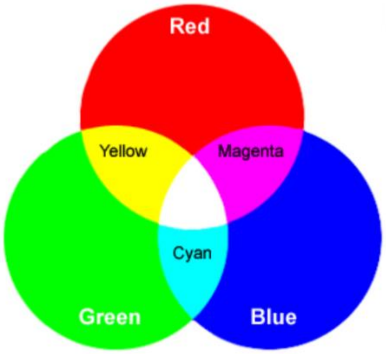
1. A light source
2. An object
3. A sensor (an eye)

Human perception of color is the sensation on the eye as the object reflects or emits light. The colors of the rainbow have traditionally been taught as being red, orange, yellow, green, blue, indigo, and violet. For humans, the actual colors in the rainbow are a mixture of just 3 colors.

Humans are trichromatic. Trichromatic vision uses three cone cells in the back of the eye that are sensitive to different wavelengths of light. There are three primary colors used to create this color space. They are additive colors of red, green, and blue.

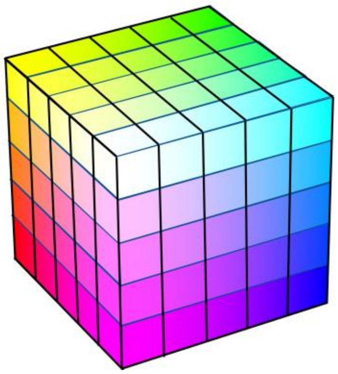
The RGB Color Space

RGB stands for Red, Green, and Blue. These are the primary colors that, when mixed together, form all the colors of the rainbow.



- When all colors are present, we see White
- Red and Green form Yellow.
- Blue and Red form Magenta
- Blue and Green form Cyan

Every color that is visible can be represented as a percentage of each of the 3 primary colors. The RGB color space is visualized as a cube. A specific color in the RGB color space is defined with three numbers that range from 0 to 255. The number represents the amount of the primary color that is present.



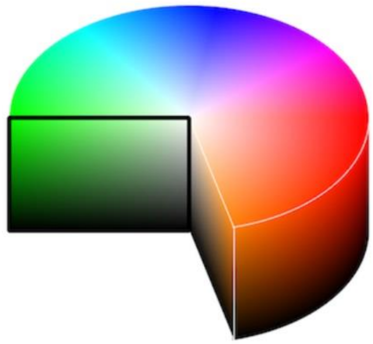
The sides are all 255 units. The corners are defined as:

White	(255,255,255)	Black	(0 , 0 , 0)
Red	(255, 0 , 0)	Yellow	(255,255, 0)
Green	(0 ,255, 0)	Magenta	(255, 0 ,255)
Blue	(0 , 0 ,255)	Cyan	(0 ,255, 255)

The system has over 256x256x256 = 16.7 million colors. The human is able to distinguish approximately 8 million.

The HCL Color Space

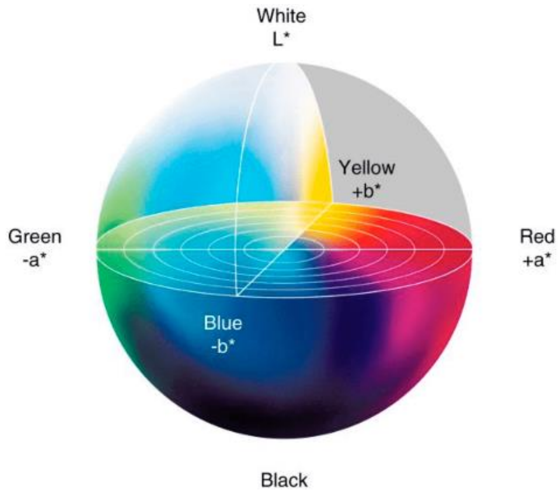
The image below shows a cylindrical representation of a color space. It is very closely related to the RGB color space.



The HCL color space above shows the change in color according to Hue, Chroma, and Luminance.

The CIE L*a*b* Color Space

CIE stands for Commission Internationale de l'Éclairage or the International Commission on Illumination. This organization has created a color space based on colors that can be perceived by the human eye. The image below shows the CIE L*a*b* color sphere.



The L-axis is the “Lightness” of the color from white to black. The a-axis is a varying degree between pure green and pure red. The b-axis is a varying degree between pure blue and pure yellow.

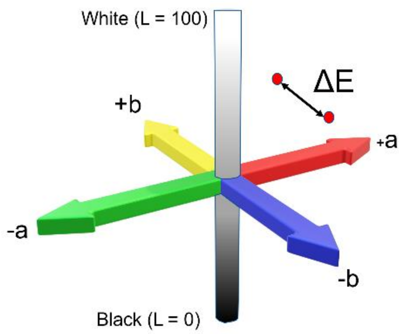
This color space is the space that is most often used to distinguish differences between colors.

The Distance Between Colors

Color matching is the technique of identifying a color and comparing it to another color within a chosen color space.

Whatever color space is used, the actual number representation of a specific color is a position within a finite 3D space. For example, a specific color within the color space can be equated to a specific coordinate within the 3D color space. If the measured colors were mathematically calculated to be close enough to each other within an acceptable predetermine limit then the colors would be considered a match. If not, then the colors would be proven by science to be dissimilar.

Therefore, the process of quantifying a match in color can be equated to a mathematical calculation. When colors are near each other in a color space it becomes difficult to distinguish a difference with the human eye.



The image above shows a distance between two points in the L*a*b* color space. This distance is called Delta E. It is a calculable distance between two points.

Delta E (ΔE) represents a calculable, reproducible, and unbiased measurement between two points that can be used to quantify if two colors are matching. Through human testing the following has been established:

ΔE < 1.0	Not perceptible by the human eye
ΔE = 1 to 2	Perceptible through close observation
ΔE = 2 to 10	Perceptible at a glance
ΔE = 11 to 49	Colors are more similar than the opposite
ΔE = 100	Colors are exactly the opposite

Example 2: Matching Siding after Fire

The two colors at the beginning of this document were from a real life situation. A house had a fire. The siding needed to be replaced. There was a question about matching. It was reported that a “reasonable” match could not be found. However, looking at different areas of the house, it became apparent that the siding had a significant fading issue. The existing siding had multi-colors. The house was not matching itself prior to the fire.

R= 118	Red = 151
G = 120	Green = 151
B =119	Blue = 149

The above colors were measured and converted to the L*a*b* CIE Color Space:

L= 50.24	L = 62.41
a = -0.96	a = -0.39
b =0.28	b = 1.05

The difference between the two colors, Delta E (ΔE), was calculated to be 11.4.

ΔE of 11.4 is much greater than 2.0; therefore, the existing siding was mathematically dissimilar. The existing colors were easily perceptible at a glance. Calculations are included at the end of this paper.

Reasoning

What was presented thus far was an introduction into the science, facts, and math related to color matching. A more difficult topic is human reasoning.

Reasoning takes knowledge and wisdom to arrive at a conclusion that is understandable. Reasoning gives a sufficient logical defense that supports or explains a decision. Reasoning requires politeness, kindness, patience, and respect to communicate with another human being.

The following sections discuss some of the issues with color matching and human reasoning.

Tolerance and the Human Observation

It is difficult to debate acceptable tolerances when the matching determination is boiled down to a yes/no answer. The amount of difference between two colors could be relaxed based on several factors.

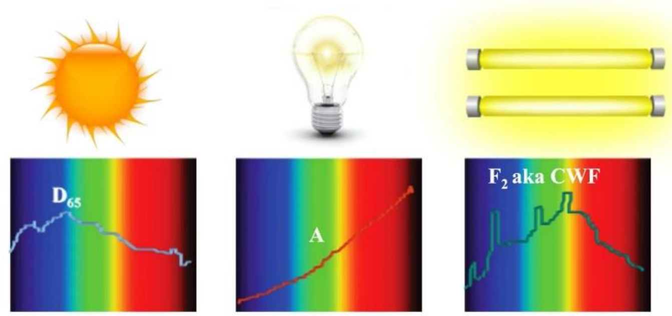
- The allowable tolerance may depend on where or what is being matched. The tolerance for dissimilar colors for a total replacement situation might be less critical compared to a patch repair situation.
- The allowable tolerance may be lessened if the dissimilar colors do not come in contact with each other or if they are completely separated from each other and never viewed together. For example, two gable end walls are never viewed side by side
- The allowable tolerance may be lessened based on the importance of the two objects being matched. An ultra-visible situation may exist where there can be no tolerance for a difference in color such as a bride’s dress and vale. Or matching color may not be critical for certain items such as trash barrels at the city dump.
- The relative amount of damage compared to the proposed amount of material to be replaced may need to be considered. This takes reasoning.

Light Source

Reasoning must be used considering that color matching may not include all the situations that will be realized. Not all light is the same. Therefore, not all reflected light will be the same.

- Noon day sunlight is different than morning sunlight which is different than sunset.
- Sunlight is different from artificial light.
- Fluorescent light is different than incandescent, halogen, or LED.

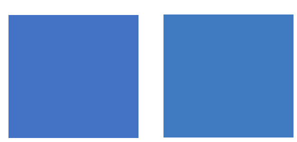
The perceived color of an object is highly dependent on the light that is being used. The image below shows the D65 light that has become a standard. As can be seen from the image the other lights have a different profile.



Fading

The image below may appear as one color, but it is not.

The above image is actually composed of five squares of slightly different shades of blue. When the two squares at the end are brought near to each other but not toughing each other, the colors may still appear the same.



When the two squares at the end are brought together and are touching each other a difference can be seen.

Additional Things to Consider

The following are offered for consideration:

1. Can a person see a noticeable color difference from locations typically traveled?
2. Can a person see a color difference only when up close in a position that is never taken by anyone other than a squirrel?
3. Is the line of sign a consideration? Can the answer on one property not be the same answer on another property due to line of site? Color matching tolerance might be different for a historical house on main street compared to a north wall of a house in the deep woods.
4. Are color matching decisions based on where the money is coming from? Are there different standards to follow if the insurance company is paying or the contractor is required to rework a mismatch, or the owner is making repairs.
5. What matching requirements are reasonable when a building was not matching prior to a loss? A repair project should never make the building worse off than it was prior to a loss, but it may be unreasonable to redo entire elevations when conditions were not matching to start with.
6. Are there distinctions to be made with regard to policy language? What has direct physical damage?
7. How is fading handled? Colors will change due to fading, weathering, and general aging. What will be the color difference going into the future?
8. If an owner made patch repairs in the past that do not match, should that penalize the prudent owner? A different owner in the past may not have allowed mismatch repairs. Should this matter?
9. Does the relative amount of damaged material compared to the whole wall or entire house play a part? Color on a large surface can mask or enhance a difference in color.

Summary

This paper did not explore other aspects of product matching such as like-quality, like-pattern, like-texture, and dimensional differences.

Each property loss ends up being unique. There are similarities, but each claim must be handled on its own merit. Minnesota case law uses the term “Reasonable.” Reasonable means no one bullies anyone, but more specifically, it means decisions need to be logical, and communicated with understanding.

The determination if two materials are matching in color does not need to be purely subjective.

When measurements and mathematics are conducted following an accepted standard, the analysis becomes unbiased, impartial, unprejudiced, neutral, and reproducible. Color matching can be brought into this realm.

However, a certain portion of color match determinations can never be based solely on science. Human reasoning is key to making the right decision.

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Calculations

Given: sRGB Find: CIE XYZ

R	=	118.00	/255	=	0.46275				
G	=	120.00	/255	=	0.47059				
B	=	119.00	/255	=	0.46667				
if	V	≤	0.04045						
then	V'	=	V	/	12.92				
else	V'	= [(	V	+0.055)/ 1.055]$^{2.4}$					
if	R	≤	0.04045						
	0.46275	≤	0.04045						
then	R_{linear}	=	R	/	12.92				
	R_{linear}	=	0.46275	/	12.92				
	R_{linear}	=	0.03582						
else	R_{linear}	= [(	R	+0.055)/ 1.055]$^{2.4}$					
	R_{linear}	= [(	0.46275	+0.055)/ 1.055]$^{2.4}$					
	R_{linear}	=	0.18116						
	R_{linear}	=	0.18116						
if	G	≤	0.04045						
	0.47059	≤	0.04045						
then	G_{linear}	=	G	/	12.92				
	G_{linear}	=	0.47059	/	12.92				
	G_{linear}	=	0.03642						
else	G_{linear}	= [(	G	+0.055)/ 1.055]$^{2.4}$					
	G_{linear}	= [(	0.47059	+0.055)/ 1.055]$^{2.4}$					
	G_{linear}	=	0.18782						
	G_{linear}	=	0.18782						
if	B	≤	0.04045						
	0.46667	≤	0.04045						
then	B_{linear}	=	B	/	12.92				
	B_{linear}	=	0.46667	/	12.92				
	B_{linear}	=	0.03612						
else	B_{linear}	= [(	R	+0.055)/ 1.055]$^{2.4}$					
	B_{linear}	= [(	0.46667	+0.055)/ 1.055]$^{2.4}$					
	B_{linear}	=	0.18447						
	B_{linear}	=	0.18447						
sR'	=	0.18116							
sG'	=	0.18782							
sB'	=	0.18447							
X_{D65}	=	0.41246	0.35758	0.18044		sR'	0.18116		
Y_{D65}	=	0.21267	0.71515	0.07218	x	sG'	0.18782		
Z_{D65}	=	0.01933	0.11919	0.9503		sB'	0.18447		
X	=	0.41246	sR'	+	0.35758	sG'	+	0.18044	sB'
X	=	0.41246	0.18116	+	0.35758	0.18782	+	0.18044	0.18447
X	=	0.17517							
Y	=	0.21267	sR'	+	0.71515	sG'	+	0.07218	sB'
Y	=	0.21267	0.18116	+	0.71515	0.18782	+	0.07218	0.18447
Y	=	0.18616							
Z	=	0.01933	sR'	+	0.11919	sG'	+	0.9503	sB'
Z	=	0.01933	0.18116	+	0.11919	0.18782	+	0.9503	0.18447
Z	=	0.2012							
X	=	17.5169							
Y	=	18.6164							
Z	=	20.1197							

Given: sRGB Find: CIE XYZ

R	=	151	/255	=	0.59216				
G	=	151	/255	=	0.59216				
B	=	149	/255	=	0.58431				
if	V	≤	0.04045						
then	V'	=	V	/	12.92				
else	V'	= [(	V	+0.055)/ 1.055]$^{2.4}$					
if	R	≤	0.04045						
	0.59216	≤	0.04045						
then	R_{linear}	=	R	/	12.92				
	R_{linear}	=	0.59216	/	12.92				
	R_{linear}	=	0.04583						
else	R_{linear}	= [(	R	+0.055)/ 1.055]$^{2.4}$					
	R_{linear}	= [(	0.59216	+0.055)/ 1.055]$^{2.4}$					
	R_{linear}	=	0.30947						
	R_{linear}	=	0.30947						
if	G	≤	0.04045						
	0.59216	≤	0.04045						
then	G_{linear}	=	G	/	12.92				
	G_{linear}	=	0.59216	/	12.92				
	G_{linear}	=	0.04583						
else	G_{linear}	= [(	G	+0.055)/ 1.055]$^{2.4}$					
	G_{linear}	= [(	0.59216	+0.055)/ 1.055]$^{2.4}$					
	G_{linear}	=	0.30947						
	G_{linear}	=	0.30947						
if	B	≤	0.04045						
	0.58431	≤	0.04045						
then	B_{linear}	=	B	/	12.92				
	B_{linear}	=	0.58431	/	12.92				
	B_{linear}	=	0.04523						
else	B_{linear}	= [(	R	+0.055)/ 1.055]$^{2.4}$					
	B_{linear}	= [(	0.58431	+0.055)/ 1.055]$^{2.4}$					
	B_{linear}	=	0.30054						
	B_{linear}	=	0.30054						
sR'	=	0.30947							
sG'	=	0.30947							
sB'	=	0.30054							
X_{D65}	=	0.41246	0.35758	0.18044		sR'	0.30947		
Y_{D65}	=	0.21267	0.71515	0.07218	x	sG'	0.30947		
Z_{D65}	=	0.01933	0.11919	0.9503		sB'	0.30054		
X	=	0.41246	sR'	+	0.35758	sG'	+	0.18044	sB'
X	=	0.41246	0.30947	+	0.35758	0.30947	+	0.18044	0.30054
X	=	0.29253							
Y	=	0.21267	sR'	+	0.71515	sG'	+	0.07218	sB'
Y	=	0.21267	0.30947	+	0.71515	0.30947	+	0.07218	0.30054
Y	=	0.30882							
Z	=	0.01933	sR'	+	0.11919	sG'	+	0.9503	sB'
Z	=	0.01933	0.30947	+	0.11919	0.30947	+	0.9503	0.30054
Z	=	0.32848							
X	=	29.253							
Y	=	30.8825							
Z	=	32.8477							

Given: CIEXYZ Find: CIELab

X	=	17.51688					
Y	=	18.61636					
Z	=	20.11967					

Reference White

Std D65	Std D50
95.0489	96.4212
100	100
108.884	82.5188

Xn	=	95.0489					
Yn	=	100					
Zn	=	108.884					
X/Xn	=	X	/	Xn			
X/Xn	=	17.51688	/	95.0489			
X/Xn	=	0.184293					
Y/Yn	=	Y	/	Yn			
Y/Yn	=	18.61636	/	100			
Y/Yn	=	0.186164					
Z/Zn	=	Z	/	Zn			
Z/Zn	=	20.11967	/	108.884			
Z/Zn	=	0.184781					
If	t	>	(6/29)^3				
Then	f(t)	=	$t^{1/3}$				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
t	=	X/Xn	=	0.184293			
If	t	>	(6/29)^3				
	0.184293	>	0.008856				
Then	f(t)	=	$t^{1/3}$				
	f(t)	=	0.184293				
	f(t)	=	0.569075				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
	f(t)	=	0.184293	/	1.241379	+	0.137931
	f(t)	=	0.28639				
	f(X/Xn)	=	0.569075				
t	=	Y/Yn	=	0.186164			
If	t	>	(6/29)^3				
	0.186164	>	0.008856				
Then	f(t)	=	$t^{1/3}$				
	f(t)	=	0.186164				
	f(t)	=	0.570994				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
	f(t)	=	0.186164	/	1.241379	+	0.137931
	f(t)	=	0.287896				
	f(Y/Yn)	=	0.570994				
t	=	Z/Zn	=	0.184781			
If	t	>	(6/29)^3				
	0.184781	>	0.008856				
Then	f(t)	=	$t^{1/3}$				
	f(t)	=	0.184781				
	f(t)	=	0.569577				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
	f(t)	=	0.184781	/	1.241379	+	0.137931
	f(t)	=	0.286782				
	f(Z/Zn)	=	0.569577				
L*	=	116	f(Y/Yn)	-	16		
L*	=	116	0.570994	-	16		
L*	=	50.23532					
a*	=	500	(	f(X/Xn)	-	f(Y/Yn))	
a*	=	500	(	0.569075	-	0.570994)	
a*	=	-0.95932					
b*	=	200	(	f(Y/Yn)	-	f(Z/Zn))	
b*	=	200	(	0.570994	-	0.569577)	
b*	=	0.283471					
L*	=	50.24					
a*	=	-0.96					
b*	=	0.28					

Given: CIEXYZ Find: CIELab

X	=	29.25305					
Y	=	30.88248					
Z	=	32.84775					

Reference White

Std D65	Std D50
95.0489	96.4212
100	100
108.884	82.5188

Xn	=	95.0489					
Yn	=	100					
Zn	=	108.884					
X/Xn	=	X	/	Xn			
X/Xn	=	29.25305	/	95.0489			
X/Xn	=	0.307768					
Y/Yn	=	Y	/	Yn			
Y/Yn	=	30.88248	/	100			
Y/Yn	=	0.308825					
Z/Zn	=	Z	/	Zn			
Z/Zn	=	32.84775	/	108.884			
Z/Zn	=	0.301677					
If	t	>	(6/29)^3				
Then	f(t)	=	$t^{1/3}$				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
t	=	X/Xn	=	0.307768			
If	t	>	(6/29)^3				
	0.307768	>	0.008856				
Then	f(t)	=	$t^{1/3}$				
	f(t)	=	0.307768				
	f(t)	=	0.675162				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
	f(t)	=	0.307768	/	1.241379	+	0.137931
	f(t)	=	0.385856				
	f(X/Xn)	=	0.675162				
t	=	Y/Yn	=	0.308825			
If	t	>	(6/29)^3				
	0.308825	>	0.008856				
Then	f(t)	=	$t^{1/3}$				
	f(t)	=	0.308825				
	f(t)	=	0.675934				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
	f(t)	=	0.308825	/	1.241379	+	0.137931
	f(t)	=	0.386707				
	f(Y/Yn)	=	0.675934				
t	=	Z/Zn	=	0.301677			
If	t	>	(6/29)^3				
	0.301677	>	0.008856				
Then	f(t)	=	$t^{1/3}$				
	f(t)	=	0.301677				
	f(t)	=	0.670678				
Else	f(t)	=	t	/	$3(6/29)^2$	+	4/29
	f(t)	=	0.301677	/	1.241379	+	0.137931
	f(t)	=	0.380948				
	f(Z/Zn)	=	0.670678				
L*	=	116	f(Y/Yn)	-	16		
L*	=	116	0.675934	-	16		
L*	=	62.4083					
a*	=	500	(	f(X/Xn)	-	f(Y/Yn))	
a*	=	500	(	0.675162	-	0.675934)	
a*	=	-0.38579					
b*	=	200	(	f(Y/Yn)	-	f(Z/Zn))	
b*	=	200	(	0.675934	-	0.670678)	
b*	=	1.051193					
L*	=	62.41					
a*	=	-0.39					
b*	=	1.05					

Given:	Color 1				Color 2			
		L^*_1	=	50.24		L^*_2	=	62.41
		a^*_1	=	-0.96		a^*_2	=	-0.39
		b^*_1	=	0.28		b^*_2	=	1.05

C^*_1	=		√	a^{*2}_1	+	b^{*2}_1
C^*_1	=		√	0.922	+	0.0784
C^*_1	=	1.000				
C^*_2	=		√	a^{*2}_2	+	b^{*2}_2
C^*_2	=		√	0.1521		1.1025
C^*_2	=	1.12				

C	=	1/2 (	C^*_1	+	C^*_2	)
C	=	1/2 (	1.00	+	1.12	)
C	=	1.0600				

G	=	0.5	[1 - √	$\bar{C}^7$	/ (	$\bar{C}^7$	+	25^7	)]
G	=	0.5	[1 - √	1.50407	/ (	1.504074	+	6.1E+09	)]
G	=	0.5	[1 - √	2.5E-10					
G	=	0.5000							

a'_1	=	1+G	a^*_1
a'_1	=	1.49999	-0.96
a'_1	=	-1.4400	
a'_2	=	1+G	a^*_2
a'_2	=	1.49999	-0.39
a'_2	=	-0.5850	

C_1'	=		√	a'^2_1	+	b^{*2}_1
C_1'	=		√	-1.43999^2	+	0.28^2
C_1'	=	1.4670				
C_2'	=		√	a'^2_2	+	b^{*2}_2
C_2'	=		√	-0.585^2	+	1.05^2
C_2'	=	1.2020				

h_1'	=	atan(	b^*_1	/	a'_1)
h_1'	=	atan(	0.28	/	-1.4400)
h_1'	=	2.950	radian		
h_1'	=	168.9964	degrees		

h_2'	=	atan(	b^*_2	/	a_2')
h_2'	=	atan(	1.05	/	-0.585)
h_2'	=	2.079	radian		
h_2'	=	119.1239	degrees		

$\Delta L'$	=	L^*_2	-	L^*_1
$\Delta L'$	=	62.41	-	50.24
$\Delta L'$	=	12.17		

$\Delta C'$	=	C_2'	-	C_1'
$\Delta C'$	=	1.20	-	1.47
$\Delta C'$	=	-0.26		

C_1'	=	1.4670	$C_1'C_2'$	=	1.76324
C_2'	=	1.2020			
h_1'	=	168.9964			
h_2'	=	119.1239			

If	$C_1'C_2'$	=	0
Then	$\Delta h'$	=	0

If	Absolute (	h_2'	-	h_1'	) <	180
If	Absolute (	119.124	-	168.996	) <	180
If		49.8725			<	180
Then	$\Delta h'$	=	h_2'	-	h_1'	
	$\Delta h'$	=	119.124	-	168.996	
	$\Delta h'$	=	-49.8725			

If	h_2'	-	h_1'	>	180			
If	119.124	-	168.996	>	180			
If		-49.8725		>	180			
Then	$\Delta h'$	=	h_2'	-	h_1'	-	360	
	$\Delta h'$	=	119.124	-	168.996	-	360	
	$\Delta h'$	=	-409.872					

If	h_2'	-	h_1'	<	-180		
If	119.124	-	168.996	<	-180		
If		-49.8725		<	-180		
Then	$\Delta h'$	=	h_2'	-	h_1'	+	360
	$\Delta h'$	=	119.124	-	168.996	+	360
	$\Delta h'$	=	310.128				

$\Delta h'$ = -49.8725

$\Delta H'$	=	$2\sqrt{(}$	C_1'	C_2'	) sin (	$\Delta h'$	/ 2)
$\Delta H'$	=	$2\sqrt{(}$	1.4670	1.2020	) sin (	-49.8725	/ 2)
$\Delta H'$	=	-1.1197					

$\bar{L}$	=	1/2 (	L_1^*	+	L_2^*	)
L	=	1/2 (	50.24	+	62.41	)
L	=	56.325				

C '	=	1/2 (	C_1'	+	C_2'	)
$\bar{C}'$	=	1/2 (	1.46696	+	1.20197	)
C '	=	1.3345				

If	Absolute (	h_1'	-	h_2'		) <	180	$C_1'C_2'$	≠	0	
	Absolute (	168.996	-	119.124		) <	180	1.76324	≠	0	
			49.8725			<	180				
then	h '	=	(	h_1'	+	h_2'	) / 2				
	h '	=	(	168.996	+	119.124	) / 2				
	h '	=	144.06								

If	Absolute (	h_1'	-	h_2'	) >	180	$C_1'C_2'$	≠	0	
	Absolute (	168.996	-	119.124	) >	180	1.76324	≠	0	
			49.8725		>	180				
If	h_1'	+	h_2'	<	360					
	168.996	+	119.124	<	360					
	288.12			<	360					
then	h '	=	h_1'	+	h_2'	+	360	) / 2		
	h '	=	168.996	+	119.124	+	360	) / 2		
	$\bar{h}'$	=	324.0602							

If	Absolute (	h_1'	-	h_2'	) >	180	$C_1'C_2'$	≠	0
	Absolute (	168.996	-	119.124	) >	180	1.76324	≠	0
			49.8725		>	180			
If	h_1'	+	h_2'	>	360				
	168.996	+	119.124	>	360				
		288.12		>	360				
then	h '	=	h_1'	+	h_2'	+	360	) / 2	
	h '	=	119.124	+	49.8725	+	360	) / 2	
	h '	=	264.498						

$\bar{h}'$ = **144.0602**

T = 1 - 0.17 cos ($\bar{h}'$ - 30) = 1.06931

+ 0.24 cos (2 $\bar{h}'$) = 0.07464

+ 0.32 cos (3 $\bar{h}'$ + 6) = 0.06555

- 0.2 cos (4 $\bar{h}'$ - 63) = 0.17858

T = **1.3881**

$\Delta\theta$ = 30 exp(-1 [$\bar{h}'$ - 275)/ 25] ^{2}

$\Delta\theta$ = 30 exp(-1 [144.060 - 275)/ 25] ^{2}

$\Delta\theta$ = 30 exp(-1 [−5.2376] ^{2}

30 exp(−27.432

$\Delta\theta$ = 0.0000

R_C = 2 √ ($\bar{C}'$ / ($\bar{C}'$ + $25'$)

R_C = 2 √ (7.53612 / (7.536122 + 6.1E+09)

R_C = 2 √ (1.2E-09)

R_C = 0.00007

$$S_L = 1 + \frac{0.015 * (L - 50)^2}{\sqrt{20 + (\bar{L} - 50)^2}}$$

$$S_L = 1 + \frac{0.015 * (56.325 - 50)^2}{\sqrt{20 + (56.325 - 50)^2}}$$

$$S_L = 1 + \frac{0.60008}{\sqrt{60.0056}}$$

S_L = **1.0775**

$$S_C = 1 + 0.045\ C'$$
$$S_C = 1 + 0.045\ 1.334464$$
$$S_C = 1.0601$$

$$S_H = 1 + 0.015\ C'\ T$$
$$S_H = 1 + 0.015\ 1.33446\ 1.3881$$
$$S_H = 1.0278$$

$$R_T = (-1)\sin(2\ \Delta\theta)\ R_C$$
$$R_T = (-1)\sin(2\ 0.000)\ 0.0001$$
$$R_T = 0.0000$$

$$k_L = 1 \quad k_C = 1 \quad k_H = 1$$

$$\Delta E^*_{00} = \sqrt{ + \left(\frac{\Delta L'}{k_L\ S_L}\right)^2 + \left(\frac{\Delta C'}{k_C\ S_C}\right)^2 + \left(\frac{\Delta H'}{k_H\ S_H}\right)^2 + R_T\ \frac{\Delta C'\ \Delta H'}{k_C\ S_C\ k_H\ S_H}}$$

$$\Delta E^*_{00} = \sqrt{ + \left(\frac{12.17}{1\ 1.0775}\right)^2 + \left(\frac{-0.26}{1\ 1.0601}\right)^2 + \left(\frac{-1.11968}{1\ 1.0278}\right)^2 + 0.0000\ \frac{-0.26\ -1.11968}{1\ 1.0601\ 1\ 1.0278}}$$

$$\Delta E^*_{00} = \sqrt{127.577 + 0.06249 + 1.18683 + -2.4E-17}$$

$$\Delta E^*_{00} = 11.4$$